

# RELIABILITY ANALYSIS OF A BRIDGE DECK UTILIZING GENERALIZED GAMMA DISTRIBUTION

This document is a technical summary of the Federal Highway Administration report, *Time-based modeling of concrete bridge deck deterioration using probabilistic models* 

## INTRODUCTION

Bridge deck safety is critical to the safety and well-being of the public; however, bridge decks continuously deteriorate over their lifetime. This process is influenced by various factors such as the attributes of the bridge deck, the surrounding environment, and the truck traffic. Pennsylvania has over 25,000 state-owned bridges and in 2019 PennDOT spent 28% of its revenue on the Highway & Bridge Maintenance program, with 29.9% of that being spent only on improvement of bridge decks (PennDOT, 2020). PennDOT performs about 18,000 bridge safety inspections every year and each bridge is inspected approximately every 2 years and assigned a condition rating (CR) for the bridge deck, superstructure, substructure, culvert, and the overall condition. This inspection dataset can be used to explore significant attributes that influence the deterioration process and develop decision-making tools for bridge deck maintenance and rehabilitation planning.

To extend the existing distribution-based deterioration models, this study adopted the generalized gamma distribution to the bridge deck reliability analysis considering the impact of covariates for a given CR. An accelerated failure time generalized gamma distribution-based (AFT-GGD) deterioration model was proposed, which can incorporate the impacts of significant attributes that influence bridge deck deterioration and can fit the data better than existing models. A corresponding Bayesian inference approach was suggested to estimate the parameters. This method allows for the estimation not only of the means of the parameters, but also the range. These ranges for the impacts of the impacts of the attributes on deterioration can be useful when determining asset management strategies considering different risk levels. This approach was demonstrated using 30 years of in-service performance data for over 22,000 bridges in Pennsylvania, and an application of updating the models as new inspection data become available was shown.

## METHODOLOGY

The goal of this research was to conduct a survival analysis of the duration that a bridge deck spends in a given CR to better account for the variability in the inspection data. For these models, the time a bridge deck lasts in each CR or cumulative truck traffic in a given CR was considered as the independent variable. These variables were modeled assuming they follow a generalized gamma distribution using an accelerated failure time approach to incorporate the covariates. Since an analytical solution was not possible, the parameters of this model were estimated using Markov Chain Monte Carlo (MCMC) methods based on Bayesian theory that can provide ranges of confidence for the results, and additionally, can update the parameters as new data become available, without re-estimating the entire model. Further, the Bayesian estimation was formulated to allow use of both uncensored data, i.e., data where the observation period covers both the beginning and the end of a condition rating, and censored data, i.e., either the beginning or the end of a condition or both is not observed.

### DATA SUMMARY

The dataset analyzed in this work consists of biannual inspections of bridges across Pennsylvania obtained from PennDOT. Historical bridge deck CR, along with attributes of the bridge structure obtained from the Bridge Management System (BMS2) (PennDOT, 2009), were accessed for over 22,000 bridge decks constructed between 1840 and 2015. These bridges were inspected between 1985 and 2015.

After cleaning the raw data, valid information for 18,354 bridges was obtained, and a total of 44,086 sojourn times were extracted and classified given the CR. In order to choose a suitable model to predict the lifecycle

performance of bridge decks, a non-parametric analysis of these sojourn times was first conducted. First, summary statistics for the distribution of the sojourn times were determined as shown in **Table 1**.

Condition Rating	Censored Count	Censored Mean (days)	Censored Std (days)	Uncensored Count	Uncensored Mean (days)	Uncensored Std (days)
CR 1	19	2,809	2,509	2	1,040	348
CR 2	104	1,690	1,263	13	1,818	1,124
CR 3	1,007	2,022	1,709	170	2,034	1,421
CR 4	3,132	3,197	2,410	783	2,581	1,717
CR 5	6,016	4,010	2,759	2,317	2,935	1,794
CR 6	7,264	4,024	2,622	3,865	2,977	1,719
CR 7	8,636	3,957	2,603	3,817	3,054	1,760
CR 8	3,234	2,610	1,927	2,612	2,501	1,443
CR 9	654	1,420	1,106	381	1,747	1,049
Total	30,066			13,960		

## Table 1. Statistic of sojourn times of bridge decks

### **EVALUATION RESULTS**

To obtain results using the proposed methodology, first the appropriate dependent variable to reflect the reliability of a bridge deck was determined. Then, the accuracy of MCMC methods was compared with maximum likelihood estimation. Next, all the attributes that were significant in predicting sojourn times were added to the model, and the influence of different attributes on the deterioration ratio were analyzed. Finally, the applicability of the Bayesian updating method was demonstrated.

## Choice of Dependent Variable

Two candidate dependent variables for modeling were possible: (1) the time until the CR changes, i.e., sojourn time; or (2) the total vehicle loading until the CR changes, i.e., the cumulative truck traffic (CTT). To evaluate the impact of different material type, how the reliability (or survival probability) would change for different bridge deck surface types was evaluated using the Kaplan-Meier estimator (Kaplan and Meier, 1958).

The results suggested that using the CTT as a dependent variable can lead to more reliable impacts of the attributes on deterioration. For example, when CTT is used as the dependent variable, epoxy overlay has the highest reliability (i.e., is expected to be stronger), followed by concrete overlay, and bituminous overlay. This is consistent with expert expectation. On the other hand, using time as the dependent variable leads to bridge decks with bituminous surface having the largest reliability compared to concrete or epoxy overlay. Similar changes in reliability for other attributes were also observed when comparing the sojourn time to the CTT as the dependent variable. Hence, the CTT was chosen as the dependent variable for this study.

### Accuracy of the MCMC Methodology

To understand the predictive power and the accuracy of the MCMC methodology, first a model with only one covariate, rebar type, was estimated. The model parameters were estimated using: (1) a maximum likelihood estimation utilizing the Newton method (Peng, 2020), which is one of the most commonly used parameter estimation methods in the literature; (2) the Metropolis-Hasting MCMC method using 20,000 samples generated from the posterior distribution. Since the MCMC method provides a distribution for each parameter, the parameters were then estimated from this method using either a maximum a posteriori estimation (MAP) or simple mean. The resulting estimates can be seen in **Figure 1** as the distribution of the parameter for the bare rebar, a normal distribution fitted to the parameter distribution, and the three point-estimates of the parameter. The results suggest that the three different point-estimates of the parameter should can closely predict the parameter values compared to standard methods. The advantage of the MCMC method is that along with the point estimation, it

provides a distribution for the parameter, which can be used to determine bands of confidence around predictions of parameters and is also useful for updating the parameters as new data become available.



Figure 1. Comparison of estimation results of bared rebar type

## Estimation of the Full Model

Next, six full models for CR 4 through CR 9 were estimated considering all attributes. To do so, all data were initially included in the model and a backwards elimination was performed. Finally, 41 attributes were included for various CR ratings. The coefficient of a given attribute represented the independent influence of this attribute, since all the attributes were incorporated into the model as binary variables (0 or 1). Hence, the model can easily be used to understand the reliability of bridges considering multiple attributes by simply summing their coefficients. For example, when a new bridge with an epoxy rebar type is constructed in District 2, and another with a bare rebar type is built in District 5, the reliability of the two bridges can be directly compared by adding the coefficients of the relevant rebar type and district number from the model to obtain the reliability of each bridge.

The coefficient estimates of the *DISTRICT* attribute suggest that bridges that had the highest reliability were in Districts 5 and 6, which are in eastern Pennsylvania. The results of the coefficient estimates for the *Physical Make-up of the Main Span of the Structure* suggest that reinforced physical type had higher reliability than pretensioned physical type in the low CRs, but pretensioned physical type had the highest reliability in CR 9. These results are consistent with empirical observations of the data, where the reliability of pretensioned concrete is the highest initially but decreases more rapidly than reinforced concrete as loading increases. This may be due to faster deterioration after initial cracking due to the progression of corrosion (Dai et al., 2020). For the *Span Interaction for the Main Span of the Structure*, model results suggest that continuous span interactions for the main span of the structure were generally stronger compared to the simple, composite span interactions. Further, composite span interactions lead to higher reliability than non-composite span interactions for most CRs, i.e., CR 4, CR 6, CR 7. For the *Rebar Type*, model results suggest that epoxy rebar had the highest reliability compared to other rebar types, which is expected. However, only 1.75% of all bridges had galvanized rebar, leading to large confidence intervals for the coefficient estimations.

### Bayesian Updating Results

As new inspection data become available, the old model will require updating. In this case, Bayesian theory can be utilized to update the parameters of the existing model. The CR 6 dataset was utilized to demonstrate the proposed method. Firstly, the dataset was divided into two parts according to the inspection date of the bridges. The first dataset consisted of all the inspection data before 2000, and the second dataset consisted of all the inspection data between 2000 and 2015. Another test that used the entire dataset as one was also developed to compare to the two-step updated results.

From the model updating results, it can be observed that as more data became available, the samples became more concentrated and the accuracy of the predictive model improved. The interval estimation also resulted in a narrower range. The updated results were very close to the direct calculations, which showed that the Bayesian updating method can obtain reliable results.

### CONCLUSION AND IMPLEMENTATION

This report suggested a new modeling approach for analyzing the influence of different attributes of bridge decks on bridge reliability. Instead of the traditional approach that considers the influence of attributes on the bridge's reliability over time, the results suggested that the use of cumulative truck traffic as the dependent variable can improve reliability of the results. A generalized gamma distribution was introduced into the infrastructure reliability analysis in this study, which is more powerful to model the bridge deck deterioration pattern, as a generalization of other commonly used distributions, such as Weibull distribution, log-normal distribution, and gamma distribution. An accelerated failure time approach was also implemented in this model to incorporate the attributes of bridge deck. A Bayesian inference-based approach was designed for the accelerated failure time generalized gamma distribution model to estimate parameters and update the model when new data become available. Issues with convergence and computational limits were also addressed. The MCMC method was utilized to get the full posterior distribution of parameters. A case study showed that the proposed method had a high accuracy and efficiency of parameters estimating and updating. The model results provided a quantitative comparison of reliabilities of bridges with different attributes, and the deterioration probability can be calculated under specific conditions. The districts where bridges are located, the rebar type, the surface overlay type, as well as other attributes were shown have a significant influence on the bridge deterioration process, and the impact of different values of these attributes was discussed.

The proposed model provides a decision-making tool to understand the influence of different attributes of the bridge on its reliability and can be used as the semi-Markov kernel to analyze the entire lifecycle deterioration process. Hence, the reliability of new bridges can be calculated and more reasonable budget allocations can be achieved to maximize the reliability of the whole infrastructure network.

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